DISTRIBUTION OF ELECTRON CONCENTRATIONS IN A DISCHARGE WITH NONLINEAR SOURCES FOR THE APPEARANCE AND DISAPPEARANCE OF PARTICLES

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We have derived an approximate solution for the one-dimensional steady equation of electron balance $D_a\nabla^2 n + \nu n + \beta n^2 = 0$, in which the coefficients ν and β may vary as to sign. The nonlinear term was approximated by two segments of straight lines by the method of least squares, as a result of which the original equation was reduced to a system of two linear differential equations whose solution is presented in analytical form. We have obtained numerical data for the determination of the electron concentration profile in the range $-8 \leq R^2\beta n_0/D_a \leq 20$ for a plane, cylindrical, and spherical configuration of the discharge zones.

The electron balance equation in a stationary positive discharge column, with consideration of ambipolar diffusion to the wall, linear and quadratic with respect to the electron concentration of the sources of formation or disappearance of particles within the volume has the form

$$D_a\nabla^2 n + \nu n + \beta n^2 = 0. \tag{1}$$

Let us rewrite Eq. (1) in dimensionless form for the one-dimensional case

$$\frac{d^2\bar{n}}{dr^2} + \frac{p}{r}\frac{\partial\bar{n}}{\partial r} + \frac{\nu R^2}{D_a}\bar{n} + \frac{\beta n_0 R^2}{D_a}\bar{n}^2 = 0, \tag{2}$$

where p is the geometry factor (p = 0 for the plane, p = 1 for the cylinder, p = 2 for the sphere); R is the radius in the case of a cylinder or sphere, or it represents the distance from the axis of symmetry to the wall in the case of plane geometry. In the average-pressure region [1] the following represents ordinary boundary conditions for Eq. (2):

$$\left.\frac{d\bar{n}}{dr}\right|_{r=0} = 0,\ \bar{n}(1) = 0. \tag{3}$$

In analytical form the solution of Eq. (2) with conditions (3) is possible only in the absence of the fourth term in the balance equation. It was initially derived by Schottky [2] for cylindrical geometry:

$$\begin{aligned} &\bar{n} = J_0(\gamma r),\ \gamma = 2{,}405 \quad \text{(cylindrical geometry)} \\ &\bar{n} = \frac{\sin(\gamma r)}{\gamma r},\ \gamma = \pi = 3{,}141 \quad \text{(spherical)} \\ &\bar{n} = \cos(\gamma r),\ \gamma = \pi/2 = 1{,}571 \quad \text{(plane)} \end{aligned} \tag{4}$$

where $\gamma^2 = \nu R^2/D_a$; J_0 is the Bessel function of the first kind, of zeroth order.

The numerical solutions of Eq. (2) have been derived for certain values of the parameter $\beta n_0 R^2/D_a$ in [3-6]. The solution of Eq. (2) in the absence of the third term is presented for cylindrical geometry in the form of elliptical functions. In [4, 5] we find an analysis of Eq. (2) and the value of the parameter $S = -\int_A (\partial n/n)dA$, is obtained numerically; here A represents the area of the lateral cross section of the discharge for all possible combinations of the parameters $\nu R^2/d_a$ and $\beta n_0 R^2/D_a$. Here we also find the normalized distributions of electron concentrations; however, we find no specific indication of the values of the above-cited parameters for which they were obtained. Finally, Eq. (2) was solved in [6] by the variational Rayleigh-Ritz method for the cases of plane and cylindrical geometry. A two-term approximation was employed. Analytical formulas are presented for the calculation of the electron concentration profile for the case

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in which $\nu > 0$ and in the range $-0.75 < \beta n_0/\nu \leq \infty$. From a brief review of the completed projects it follows that the solutions of Eq. (2) have been obtained for certain special cases or in certain intervals of change in the determining parameters. It therefore remains urgent subsequently to develop additional methods for the solutions of Eq. (2), and here, from the practical standpoint, it would be desirable to achieve these in the form of analytical approximations. It is precisely to this problem that the present study is devoted.

Initially we will examine the case involving cylindrical geometry, i.e., with p = 1. The nonlinear function $\bar{n}^2$ changes from zero at the wall of the discharge chamber (r = 1) to 1 at the axis of symmetry (r = 0). In this interval we will approximate the function $\bar{n}^2$ with two broken lines.

$$\bar{n}^2 = \begin{cases} a\bar{n}, & 0 \leqslant \bar{n} \leqslant k, \quad r_1 \leqslant r \leqslant 1, \\ 1 + b(\bar{n} - 1), & k \leqslant \bar{n} \leqslant 1, \quad 0 \leqslant r \leqslant r_1, \end{cases} \tag{5}$$

where $ak = 1 + b(k - 1)$. Approximation by the method of least squares yields the following values for the coefficients in formula (5): $a = 0.375$, $b = 1.625$, $k = 0.5000$. Substituting $\bar{n}^2$ in Eq. (2) with the approximating expression (5), we arrive at the following system of two linear equations:

$$\begin{gathered} \frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \left(\frac{\nu R^2}{D_a} + b\frac{\beta n_0 R^2}{D_a}\right)\bar{n} + \frac{\beta n_0 R^2}{D_a}(1-b) = 0, \quad 0 \leqslant r \leqslant r_1; \\ \frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \left(\frac{\nu R^2}{D_a} + a\frac{\beta n_0 R^2}{D_a}\right)\bar{n} = 0, \quad r_1 \leqslant r \leqslant 1. \end{gathered} \tag{6}$$

For the sake of convenience, we will denote the subsequent notation as follows:

$$\left(\frac{\nu R^2}{D_a} + b\frac{\beta n_0 R^2}{D_a}\right) = \gamma^2; \quad \left(\frac{\nu R^2}{D_a} + a\frac{\beta n_0 R^2}{D_a}\right) = \lambda^2; \quad \frac{\beta n_0 R^2}{D_a} = T;$$

$$\frac{\nu R^2}{D_a} = M; \quad (1-b) = C.$$

Equations (6) can then be represented in the following form:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \gamma^2\bar{n} + TC = 0, \quad 0 \leqslant r \leqslant r_1, \tag{7a}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \lambda^2\bar{n} = 0, \quad r_1 \leqslant r \leqslant 1. \tag{7b}$$

The function $\bar{n}$ must satisfy condition (3), as well as the conditions of conjugacy for the solutions of Eqs. (7a) and (7b) for $r = r_1$, i.e., at this point the equality of electron concentrations and their first derivatives must be satisfied. The general solutions of Eqs. (7a) and (7b), with consideration of the first of the conditions in (3), are written in the form

$$\bar{n} = \begin{cases} C_1 J_0(\gamma r) - TC/\gamma^2, & 0 \leqslant r \leqslant r_1, \\ C_2 J_0(\lambda r) + C_3 Y_0(\lambda r), & r_1 \leqslant r \leqslant 1, \end{cases} \tag{8}$$

where Y_0 is the Bessel function of the second kind, of zeroth order; C_1, C_2, and C_3 are coefficients.

It follows from the conditions $\bar{n}(0) = 1$ and $\bar{n}(1) = 0$ that

$$C_1 = 1 + TC/\gamma^2, \tag{9}$$

$$C_2 J_0(\lambda) + C_3 Y_0(\lambda) = 0. \tag{10}$$

The conjugacy conditions for $r = r_1$ yield the following equations:

$$C_1 J_0(\gamma r_1) - TC/\gamma^2 = k, \tag{11}$$

$$C_2 J_0(\lambda r_1) + C_3 Y_0(\lambda r_1) = k, \tag{12}$$

$$C_1 \gamma J_1(\gamma r_1) = C_2 \lambda J_1(\lambda r_1) + C_3 \lambda Y_1(\lambda r_1), \tag{13}$$

where J_1 and Y_1 are first-order Bessel functions, of the first and second kind, respectively. Thus we have five unknowns: C_1, C_2, C_3, γ, and r_1 and as many equations (9)-(13). In these equations T is a specified parameter, while λ is expressed in terms of γ: $\lambda^2 = \gamma^2 + T(a - b)$. After transformation, we reduce system (9)-(13) to two equations with two unknowns γ and r_1:

$$J_0(\gamma r_1) = (k + TC/\gamma^2)/(1 + TC/\gamma^2), \tag{14}$$

$$(1 + TC/\gamma^2)\,\gamma J_1(\gamma r_1) = k\lambda\,[J_0(\lambda)\,Y_1(\lambda r_1) - Y_0(\lambda)\,J_1(\lambda r_1)]/Q, \tag{15}$$

where $Q = J_0(\lambda)Y_0(\lambda r_1) - Y_0(\lambda)J_0(\lambda r_1)$. System (14), (15) was solved numerically in its dependence on the parameter T. Subsequently, based on the formulas

$$C_2 = -kY_0(\lambda)/Q;\ C_3 = kJ_0(\lambda)/Q \tag{16}$$

we calculated the coefficients in formula (8). The coefficient C_1 for all cases (p = 0, p = 1, p = 2) is calculated in accordance with formula (9). The positiveness of $\bar{n}$ is tested over the entire region of its determination.

Analogous results were obtained for spherical (p = 2) and plane (p = 0) geometries. For the case p = 2 the formula for the calculation of electron concentration is written in the form

$$\bar{n} = \begin{cases} C_1 \dfrac{\sin(\gamma r)}{\gamma r} - TC/\gamma^2, & 0 \leqslant r \leqslant r_1, \\ C_2 \sin(\lambda r)/\lambda r + C_3 \cos(\lambda r)/\lambda r, & r_1 \leqslant r \leqslant 1, \end{cases} \tag{17}$$

where γ and r_1 are calculated from the system of equations

$$\sin(\gamma r_1)/\gamma r_1 = (k + TC/\gamma^2)/(1 + TC/\gamma^2), \tag{18a}$$

$$(1 + TC/\gamma^2)[r_1 \cos(\gamma r_1) - \sin(\gamma r_1)/\gamma] = -k\lambda r_1\,\{r_1 \operatorname{ctg}[\lambda(1 - r_1)] + 1/\lambda\}, \tag{18b}$$

while the coefficients C_2 and C_3 are calculated on the basis of the following formulas:

$$C_2 = -k\lambda \cos(\lambda)/\sin[\lambda(1 - r_1)],\ C_3 = k\lambda \sin(\lambda)/\sin[\lambda(1 - r_1)]. \tag{19}$$

For plane geometry (p = 0) the profile of electron concentration is calculated on the basis of the following formula:

$$\bar{n} = \begin{cases} C_1 \cos(\gamma r) - TC/\gamma^2, & 0 \leqslant r \leqslant r_1, \\ C_2 \cos(\lambda r) + C_3 \sin(\lambda r), & r_1 \leqslant r \leqslant 1, \end{cases} \tag{20}$$

where γ and r_1 are calculated from the system of equations

$$\cos(\gamma r_1) = (k + TC/\gamma^2)/(1 + TC/\gamma^2), \tag{21}$$

$$\gamma(1 + TC/\gamma^2) \sin(\gamma r_1) = k\lambda \operatorname{ctg}[\lambda(1 - r_1)], \tag{22}$$

while the coefficients C_2 and C_3 are calculated on the basis of the following formulas:

$$C_2 = k \sin(\lambda)/\sin[\lambda(1 - r_1)],\ C_3 = -k \cos(\lambda)/\sin[\lambda(1 - r_1)]. \tag{23}$$

The results of the calculations can be found in Table 1. Figure 1 shows some of the electron-concentration profiles that have been calculated for cylindrical and spherical geometries. In the particular case in which $T \to 0$, the results of the calculation coincide with the familiar analytical solution (4). This can e seen from the computational data for T = -0.01. The negative values of T, and this thus included β, correspond to a discharge regime with predominant volume recombination. With an increase in $|T|$ in this region the electron concentration profile becomes flatter and flatter, which corresponds to a reduction in the parameter γ. The electron concentration gradient becomes larger only near the wall. Consequently,

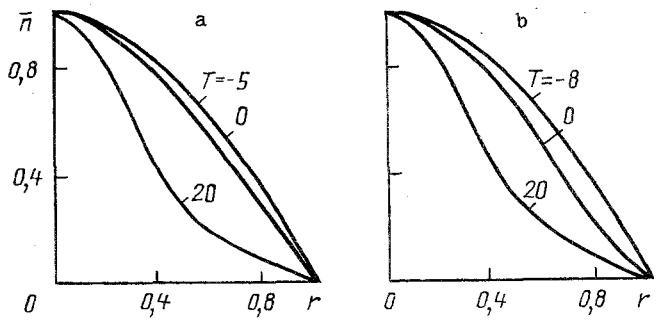

Fig. 1. Distributions of dimensionless electron concentrations: a) p = 1; b) p = 2.

TABLE 1. Coefficients in Formulas (8), (17), and (20) as a Function of the Parameter T

T	M	C_1	C_2	C_3	γ	λ	r_1
			Cylinder	$(p=1)$			
—5	9,453	3,352	1,074	3,963	1,153	2,753	0,882
—2	7,151	1,320	1,083	1,387	1,975	2,530	0,872
—0,01	5,778	0,999	0,999	—0,0003	2,407	2,405	0,858
2	4,548	0,840	0,890	—0,094	2,792	2,302	0,839
5	2,932	0,717	0,730	—0,161	3,325	2,193	0,805
10	0,824	0,634	0,540	—0,150	4,132	2,139	0,742
20	—1,917	0,591	0,369	—0,016	5,530	2,363	0,625
			Plane	$(p=0)$			
—3	5,043	12,128	0,848	0,367	0,410	1,980	0,897
—2	4,173	2,354	0,906	0,260	0,961	1,850	0,892
2	0,818	0,693	1,059	—0,349	2,017	1,252	0,865
5	—1,527	0,526	1,075	—1,607	2,569	0,590	0,835
			Sphere	$(p=2)$			
—8	14,686	3,965	1,391	—0,395	1,299	3,419	0,882
—5	12,686	1,685	1,260	—0,186	2,136	3,288	0,868
—2	10,907	1,163	1,105	—0,050	2,767	3,187	0,850
2	8,971	0,898	0,900	0,021	3,496	3,118	0,820
5	7,907	0,805	0,766	0,011	4,004	3,128	0,792
10	6,880	0,730	0,598	—0,071	4,809	3,260	0,739
20	6,683	0,681	0,379	—0,273	6,260	3,766	0,645

this method is constrained in the case of rather large negative T, a limitation imposed by the absence of solutions for Eq. (15) which satisfy the condition of conjugacy with respect to the first derivative. As T increases toward the positive values which coincide to discharge with predominant stepwise ionization ($\beta > 0$), the electron concentration distribution becomes increasingly compressed against the discharge axis. In mathematical terms this is equivalent to an increase in the parameter γ. In this case, within some range of variations in T the parameter M, and this therefore also includes ν, remains positive. This corresponds to the discharge regime in which, in addition to the stepwise ionization, direct ionization exists as well. In the case of sufficiently large positive T for the cylinder and the plane M and γ become negative. This discharge regime is characterized by the predominance of stepwise ionization processes ($\beta > 0$) and electron adhesion ($\nu < 0$).

For the plane geometry let us compare the calculational results obtained from formulas (20) to the solution which in [6] is approximated by the formula

$$\bar{n} = C_4 \cos\left(\frac{\pi}{2} r\right) + C_5 \cos\left(\frac{3\pi}{2} r\right), \tag{24}$$

where

$$C_5/C_4 \approx 0{,}0212 \left(\frac{\beta n_0/\nu}{1 + 0{,}812\beta n_0/\nu}\right).$$

For T = -3 and M = 5.043 the value of $\beta n_0/\nu$ = T/M = -0.595 and C_5/C_4 = -0.0244. When we take into consideration that $C_4 + C_5 = 1$, in this case (24) is written in the form

$$\bar{n} = 1{,}02501 \cos\left(\frac{\pi}{2} r\right) - 0{,}02501 \cos\left(\frac{3\pi}{2} r\right). \quad (25)$$

Comparison of the calculations based on formulas (20) and (25) demonstrates that up to values of r = 0.5 there is virtually no divergence (<1%), while it increases in the range $0.5 < r \leq 1$, not exceeding 6%. In this case the values of $\bar{n}$ are more readily found from formula (20). Let us note that in comparison with [6] the approximate solution of Eq. (2) has also been found for the region with a negative value for the parameter ν.

NOTATION

n, electron concentration; n_0, electron concentration at the axis of symmetry; $\bar{n} = n/n_0$, dimensionless electron concentration; x, coordinate; R, characteristic dimension of discharge zone; r = x/R, dimensionless coordinate; D_a, coefficient of ambipolar diffusion; ν, effective coefficient of direct ionization ($\nu > 0$) or adhesion ($\nu < 0$); β, effective coefficient of stepwise ionization ($\beta > 0$) or recombination ($\beta < 0$).

LITERATURE CITED

1. Yu. B. Golubovskii and R. I. Lyagushenko, *Inzh.-Fiz. Zh.*, **46**, No. 11, 2327-2332 (1976).
2. W. Schottky, *Phys. Z.*, **25**, 342-348 (1924).
3. E. Spenke, *Phys. Z.*, **127**, 221-228 (1950).
4. G. L. Rogoff, *J. Appl. Phys.*, **50** (11), 6806-6810 (1979).
5. G. L. Rogoff, *J. Appl. Phys.*, **51** (6), 3144-3148 (1980).
6. F. W. Crawford, *J. Appl. Phys.*, **51** (3), 1422-1430 (1980).

POSSIBILITIES OF ELEVATING SPECIFIC CHARACTERISTICS OF ACTIVE MEDIA WITH HEAT PUMPING AT LOW CO_2 CONCENTRATIONS

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We present the calculational results, as well as those of our experimental study, with respect to the specific characteristics of active media with a low carbon-dioxide content.

In the works of numerous authors, including [1, 2], reference is unavoidably made to the influence exerted by the composition, temperature, and pressure of the gas on the characteristics of carbon-dioxide-based molecular lasers. The expansion of heated mixtures of CO_2 and H_2O with nitrogen in a supersonic nozzle is a well-known method of achieving markedly nonequilibrium media. The drawback of this method is its comparatively high relaxation losses, generally amounting to 50-60% at a carbon-dioxide gas concentration of Ψ_C = 10-20% and a water vapor concentration of Ψ_H = 0.5-3%. The utilization of nozzles in which the components are mixed eliminates this shortcoming, but some of the positive characteristics of the homogeneous method are lost. There exists an alternative possibility for reducing the relaxation losses, namely to utilize gas mixtures with a limited CO_2 content at a level of 1-3%. Heat-exchange heaters made on a base of aluminum or zirconium ceramics [3-5] allow us to heat the working mixture to a temperature of 2300-2500 K. The calculations and experiments carried out by the authors demonstrate that under the above-described conditions the relaxation losses do not exceed 15-25%, while the specific disposable energy (the energy stored in the oscillations of the nitrogen molecules and the antisymmetric oscillation mode of the CO_2, multiplied by the quantum efficiency) amounts to 50-100 J/g at a deceleration pressure of 2.5-6 MPa.

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